Available online at www.sciencedirect.com
ScienceDirect
Nuclear Instruments and Methods in Physics Research A 583 (2007) 479–484
NUCLEAR INSTRUMENTS & METHODS IN PHYSICS RESEARCH Section A
www.elsevier.com/locate/nima

Optimization of the data taking strategy for a high precision τ mass measurement ☆

Y.K. Wang[a,b], X.H. Mo[a,*], C.Z. Yuan[a], J.P. Liu[b]

[a] Institute of High Energy Physics, Chinese Academy of Sciences, Beijing 100049, China
[b] School of Physics Science and Technology, Wuhan University, Wuhan 430072, China

Received 2 July 2007; received in revised form 2 September 2007; accepted 17 September 2007
Available online 29 September 2007

Abstract

To achieve a high precision τ mass (m_τ) measurement at the forthcoming high luminosity experiment, Monte Carlo simulation and sampling technique are adopted to simulate various data taking cases from which the optimal scheme is determined. The study indicates that when m_τ is the sole parameter to be fit, the optimal energy for data taking is located near the $\tau^+\tau^-$ production threshold in the vicinity of the largest derivative of the cross-section to energy; one point in the optimal position with luminosity around $63\,\text{pb}^{-1}$ is sufficient for getting a statistical precision of $0.1\,\text{MeV}/c^2$ or better.

PACS: 14.60.Fg; 02.60.Pn; 02.70.Uu

Keywords: τ mass; Statistical optimization; e^+e^- annihilation

1. Introduction

The simplest and most precise technique to determine the mass of the τ lepton (m_τ) is a scan of the $\tau^+\tau^-$ cross-section near the production threshold in an e^+e^- annihilation. This technique needs a precise calibration of the beam energy and a good understanding of the production cross-section near threshold. Experimentally, depolarization technique has been developed by KEDR collaboration to realize a high accurate determination of the beam energy at a level of 10^{-6} [1], while theoretically, precision at a level of 10^{-4} has been achieved for the $\tau^+\tau^-$ production cross-section near the threshold [2–4]. Moreover, large τ data sample is expected from the forthcoming experiment BESIII [5], therefore it is of great interest to know what precision one expects to achieve in m_τ measurement in the near future.

For the BES collaboration, a fairly high precise value of m_τ has been achieved by the threshold scan method [6–8]:

$$m_\tau = 1776.96^{+0.18+0.25}_{-0.21-0.17}\,\text{MeV}/c^2, \tag{1}$$

with a relative error of 10^{-4}. We note that the relative statistical (1.6×10^{-4}) and the systematic (1.7×10^{-4}) uncertainties are at the same level. In this work, we would focus on the statistical aspect of τ mass measurement at BESIII. So far as the effect due to systematic uncertainties is concerned, we merely present a preliminary estimations at the end of paper.

Herein to achieve high precision in m_τ we want to find out:

1. What is the optimal distribution (position) of the data taking points?
2. How many energy points are needed for the scan in the vicinity of threshold?
3. How much luminosity is required for certain precision expectation?

2. Methodology

Within a specified period of data taking time or equivalently for a given integrated luminosity, we try to

☆Supported by National Natural Science Foundation of China (10491303) and 100 Talents Program of CAS (U-25).
*Corresponding author. Tel.: +86 10 88236317; fax: +86 10 88233083.
E-mail address: moxh@mail.ihep.ac.cn (X.H. Mo).

0168-9002/$ - see front matter © 2007 Elsevier B.V. All rights reserved.
doi:10.1016/j.nima.2007.09.026

find out the scheme which can provide the highest precision on m_τ. The sampling technique is utilized to simulate various data taking possibilities among which the optimal one is chosen. For certain simulation, the likelihood function is constructed as follows [6–8]:

$$\mathrm{LF}(m_\tau) = \prod_i^{N_{\mathrm{pt}}} \frac{\mu_i^{N_i} \mathrm{e}^{-\mu_i}}{N_i!}, \tag{2}$$

where N_i is the observed number of $\tau^+\tau^-$ events for $\mathrm{e}\mu$ final state[1] at scan point i; μ_i is the expected number of events and given by

$$\mu_i(m_\tau) = [\varepsilon \cdot B_{\mathrm{e}\mu} \cdot \sigma_{\mathrm{obs}}(m_\tau, E^i_{\mathrm{cm}}) + \sigma_{\mathrm{BG}}] \cdot \mathscr{L}_i. \tag{3}$$

In Eq. (3), $\mathscr{L}_i$ is the integrated luminosity at the ith point; ε is the overall efficiency of $\mathrm{e}\mu$ final state for identifying $\tau^+\tau^-$ events, which includes trigger efficiency and event selection efficiency; $B_{\mathrm{e}\mu}$ is the combined branching ratio for decays $\tau^+ \to \mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\tau$ and $\tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$, or the corresponding charge conjugate mode; σ_{obs}, which can be calculated by the improved Voloshin's formulas [2], is the observed cross-section measured at energy E^i_{cm} with m_τ as a parameter; and σ_{BG} is the total cross-section of background channels after $\tau^+\tau^-$ selection.

In the following study concerned with statistical uncertainty, we take $\varepsilon = 14.2\%$ [9], Δ (energy spread[2]) $= 1.4\,\mathrm{MeV}$, $B_{\mathrm{e}\mu} = 0.06194$ [10], and neglect corresponding uncertainties whose effects will be discussed briefly in Section 5. As to σ_{BG}, the previous experience [6] indicates that $\sigma_{\mathrm{BG}} \approx 0.024\,\mathrm{pb}$ which is fairly small comparing with the $\tau^+\tau^-$ production cross-section (0.1 nb) near threshold. Moreover, at the forthcoming detector with high luminosity, a large data sample can be taken below the threshold to measure σ_{BG} rather accurately. In actual fit as a constant, σ_{BG} has tiny effect on the optimization of points distribution. For simplification, σ_{BG} is set to be zero, which means that our study is background free.

In the following analysis, the value of τ mass itself is assumed to be known and under such an assumption, we attempt to answer the questions proposed at the end of the previous section. Nevertheless, when think twice about the first two questions, we observe that they actually intertwist with each other, that is the optimal number of point depends on the distribution of points and *vice versa*. To resolve such a dilemma, we start from a simple distribution and find the optimal number of points, then based on which we finally determine the number of points.

[1] For briefness, the $\mathrm{e}\mu$ channel ($\tau^+ \to \mathrm{e}^+\nu_\mathrm{e}\bar{\nu}_\tau, \tau^- \to \mu^-\bar{\nu}_\mu\nu_\tau$, or $\tau^+ \to \mu^+\nu_\mu\bar{\nu}_\tau, \tau^- \to \mathrm{e}^-\bar{\nu}_\mathrm{e}\nu_\tau$) is considered firstly; the statistical significance will be further improved when more channels are taken into account, see Section 5 for detailed discussion.

[2] In the simulation, the energy varies in a range, so the energy spread is actually calculated by an empirical formula: $\Delta = (0.16203E^2_{\mathrm{cm}}/4 + 0.89638) \times 10^{-3}\,\mathrm{GeV}$, which gives $\Delta = 1.4\,\mathrm{MeV}$ at $E_{\mathrm{cm}}/2 = 1.77699\,\mathrm{GeV}$ [9].

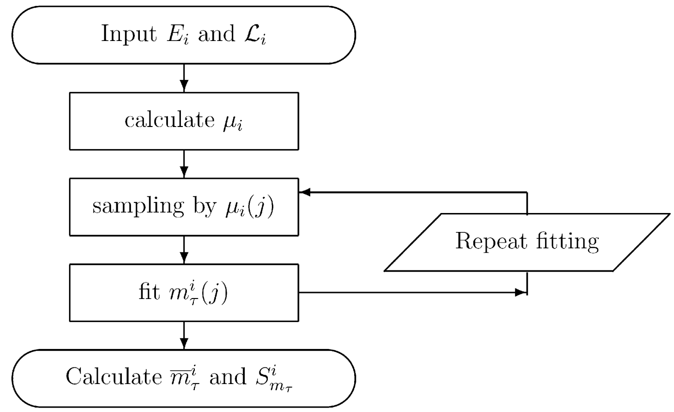

Fig. 1. Flow chart of sampling simulation, where i ($i = 1, 2, \ldots, N_{\mathrm{pt}}$) indicates certain scheme and j ($j = 1, 2, \ldots, N_{\mathrm{samp}}$) sampling times.

3. First optimization

As a tentative beginning, the energy interval to be studied is divided evenly, viz.

$$E_i = E_0 + (i-1) \times \delta E \quad (i = 1, 2, \ldots, N_{\mathrm{pt}}) \tag{4}$$

where the initial point $E_0 = 3.545\,\mathrm{GeV}$, the final point $E_\mathrm{f} = 3.595\,\mathrm{GeV}$, and the fixed step $\delta E = (E_\mathrm{f} - E_0)/N_{\mathrm{pt}}$ with N_{pt} being the number of energy points. For a given integrated luminosity ($\mathscr{L}_{\mathrm{tot}}$) it is also apportioned averagely at each point[3], i.e. $\mathscr{L}_i = \mathscr{L}_{\mathrm{tot}}/N_{\mathrm{pt}}$.

In order to reduce the statistical fluctuation, sampling is repeated many times ($N_{\mathrm{samp}} = 500$ for our study) for each scheme (say for each N_{pt}), the average value and corresponding variance of the fit out τ mass are worked out as follows [11]:

$$\overline{m}^i_\tau = \frac{1}{N_{\mathrm{samp}}} \sum_{j=1}^{N_{\mathrm{samp}}} m^i_{\tau j}, \tag{5}$$

$$S^2_{m_\tau}(m^i_\tau) = \frac{1}{N_{\mathrm{samp}} - 1} \sum_{j=1}^{N_{\mathrm{samp}}} (m^i_{\tau j} - \overline{m}^i_\tau)^2. \tag{6}$$

Here it should be noted that i indicates the certain scheme, whose value can be 1 while j indicates the sampling times which equals to 500 in the following study. Without special declaration, the meaning of the average defined by Eqs. (5) and (6) will be kept in the study follows.

The general flow chart of sampling and fitting research is presented in Fig. 1.

3.1. Data taking points

Using the experiment parameters given in Section 2, ε, Δ, and $B_{\mathrm{e}\mu}$, setting $\mathscr{L}_{\mathrm{tot}} = 30\,\mathrm{pb}^{-1}$, and N_{pt} ranging from 3 to 20, the fitted results are shown in Fig. 2(a), or more clearly

[3] Another scheme is to apportion the total number of event evenly at each point, the luminosity at each point is determined by relation $\mathscr{L}_i = \mathscr{L}_{\mathrm{tot}}/(\sigma_i \cdot \sum_i 1/\sigma_i)$ with σ_i denotes the cross-section at the ith point. Such a scheme leads to the same final conclusion as that depicted in this work.

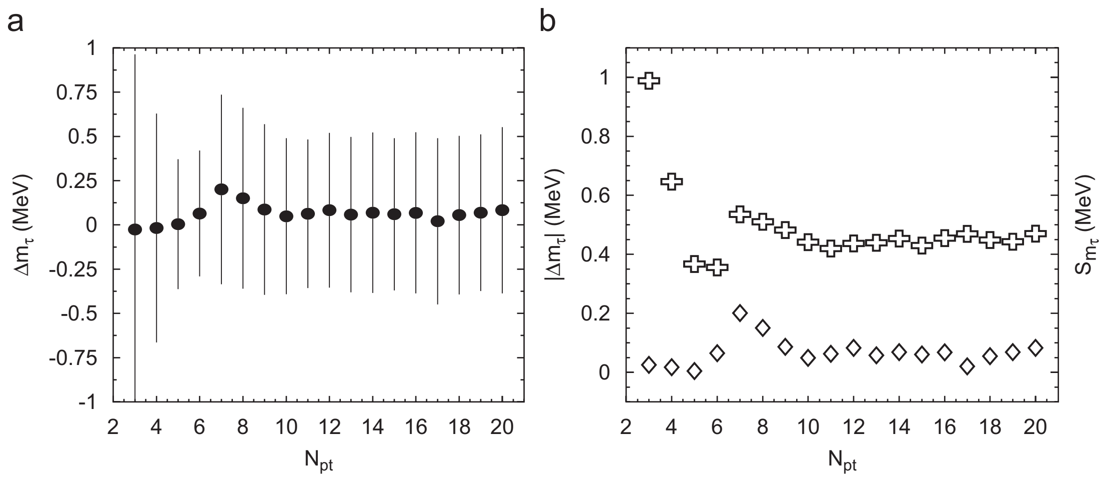

Fig. 2. The variation of Δm_τ (or $|\Delta m_\tau|$) and S_{m_τ} against the number of points N_{pt}. In (a) the dots and error bars represent Δm_τ and S_{m_τ}, respectively; in (b) the diamonds denote $|\Delta m_\tau|$ and the crosses S_{m_τ}.

Table 1
The relation between luminosity and statistical uncertainty of m_τ

$\mathscr{L}_{tot}$ (pb^{-1})	$E_{cm} = 3.55398\,GeV$		$E_{cm} = 3.5548\,GeV$	
	Sm_τ (MeV)	Δm_τ (MeV)	Sm_τ (MeV)	Δm_τ (MeV)
9	0.24874	0.02931	0.29240	0.02114
18	0.16926	0.01550	0.19635	0.00756
27	0.14024	0.01234	0.15670	0.00475
36	0.12130	0.00812	0.14384	0.00504
45	0.10653	0.00824	0.12717	0.00292
54	0.09783	0.00717	0.10714	−0.00037
63	0.09035	0.00726	0.09923	−0.00003
72	0.08424	0.00520	0.09297	0.00008
100	0.06781	0.00129	0.07876	−0.00002
1000	0.02146	0.00016	0.02515	0
10 000	0.00684	0	0.00805	0

in Fig. 2(b), where $\Delta m_\tau = \overline{m}_\tau - m_\tau^0$, the difference between the fitted m_τ and the input one ($m_\tau^0 = 1776.99\,MeV/c^2$ according to PDG06 [10]), and S_{m_τ} the corresponding uncertainty.

The Δm_τ here reflects the fit bias due to the Poisson distribution for the small number of events. With the increasing luminosity of data taking (correspondingly the increasing of the number of events), the bias also tends to zero (wherein the Poisson approximates a Gaussian distribution). This point can be seen more clearly in the following study (refer to Table 1 for the variation of Δm_τ with the increasing of luminosity).

For the uncertainty of the fit two points should be noted: first, S_{m_τ} is much larger than $|\Delta m_\tau|$ (the absolute value of Δm_τ), so from the point view of accuracy, the former is much more crucial than the latter. Therefore, S_{m_τ} is adopted to assess the quality of fit, in another word, the smaller the S_{m_τ} the better is the fit. Second, it is prominent that too few data taking points lead to large uncertainty while too many points have no contribution for precision improvement either. As indicated in Fig. 2(b), $N_{pt} = 5$ is taken as the optimized number of points for the evenly-divided-distribution scheme.

3.2. Optimal distribution

With five points, we want to further search for the distribution which can afford us the small fit uncertainty. As without any theoretical or empirical guidance, various possibilities are tried by employing the sampling technique, that is the energy points are taken randomly in the chosen interval. For 200 times sampling, singled out are two fit results with the smallest ($S_{m_\tau} = 0.152\,MeV/c^2$, denoted by stars) and greatest ($S_{m_\tau} = 1.516\,MeV/c^2$, denoted by diamonds) fit uncertainties; their distributions are shown in Fig. 3, by virtue of which it is obvious when the points crowd near the threshold the uncertainty is small; on the contrary, when the points are far from the threshold the uncertainty becomes large.[4] More mathematically, it is found that the smallest uncertainty is acquired when points gather at the region with the large derivative of the cross-section to energy. So this result implies that the region with large derivative is presumably the optimal position for data taking. This speculation will be proved in the next section.

4. Second optimization

Based on the above studies, we are to find out (a) the region which is sensitive to fit uncertainty, (b) the optimal number of points in the region, and (c) the position for the optimal points.

4.1. Optimal region

To hunt for the sensitive position, two regions are selected as shown in Fig. 4(a): the region I ($E_{cm} \subset$

[4]The first point for two kinds of fit in Fig. 3 is actually ineffective for estimator in Eq. (2) since the expected number of events equals to zero ($\mu = 0$) below threshold.

(3.553, 3.558) GeV) is selected with the derivative falls to 75% of its maximum while the region II ($E_{cm} \subset$ (3.565, 3.595) GeV) is selected with the variation of derivative is comparative smooth than that in region I.

To confirm the fore-mentioned speculation, two schemes are designed. For the first scheme, two points are taken in the region I, one at 3.55398 GeV as the threshold point and the other at 3.55484 GeV corresponding to the largest derivative point. As in the region II, the number of points N_{pt} increases from 1 to 20, with each point having the luminosity $5\,pb^{-1}$. The fit results are displayed in Fig. 4(b) by the crosses. Clearly, the increase of points in the region II hardly has contribution for accuracy improvement ($S_{m_\tau} = 0.25\,MeV/c^2$ almost remains the same with the increasing number of points in region II). That is to say, the region I is the sensitive region so far as fit uncertainty is concerned while the region II is not. To prove this point further, for the second scheme, merely the points in the region II are taken, N_{pt} also increases from 1 to 20. The fit results are displayed in Fig. 4(b) by the diamonds. As expected, with the increasing number of points, S_{m_τ} decreases as well, but even with 20 points in the region II the value of $S_{m_\tau} = 0.73\,MeV/c^2$ is still much larger than that with solely two points in the region I. So it is concluded that the points within the region I are more useful for optimal data taking.

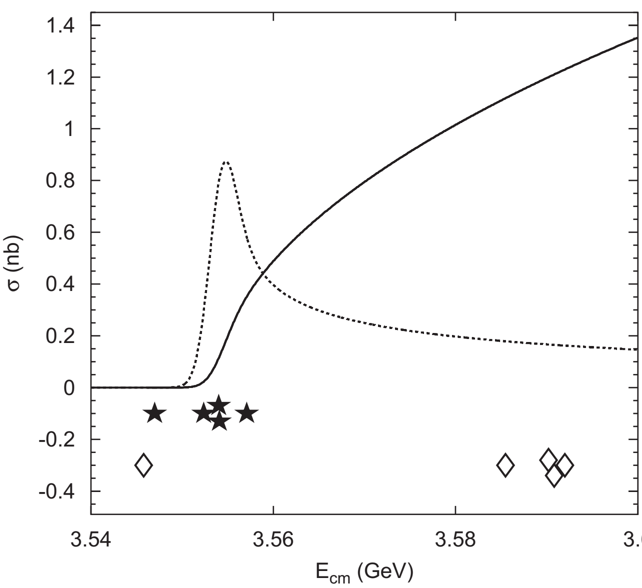

Fig. 3. The distributions of the data taking points with the smallest (denoted by stars) and greatest (denoted by diamonds) S_{m_τ}. The solid curve is the calculated observed cross-section, and the dashed line the corresponding derivative of cross-section to energy with a scale factor 10^{-2}.

4.2. Optimal position

In this subsection, the first thing needed to be known is how many points are optimal in the region with large derivative. As the procedure in Section 3.1, the total luminosity $\mathscr{L}_{tot} = 45\,pb^{-1}$ is rationed averagely into N_{pt} points ($N_{pt} = 1, 2, \ldots, 6$) within the energy region from 3.553 to 3.557 GeV. The results are shown in Fig. 5, according to which the number of points has weak effect on the final uncertainty. In other words, within the large derivative region, one point suffices to give rise to small uncertainty. This is easy to understand since there is only one free parameter (m_τ) needed to be fit in the $\tau^+\tau^-$ production cross-section, one measurement will further fix the normalization of the curve. The larger the derivative, the more sensitive to the mass of the τ lepton.

Since one point is enough, then an immediate question is where the optimal point should locate? To answer it, the scan with one point with the luminosity $\mathscr{L}_{tot} = 45\,pb^{-1}$ is made and the results are shown in Fig. 6. Just as previous

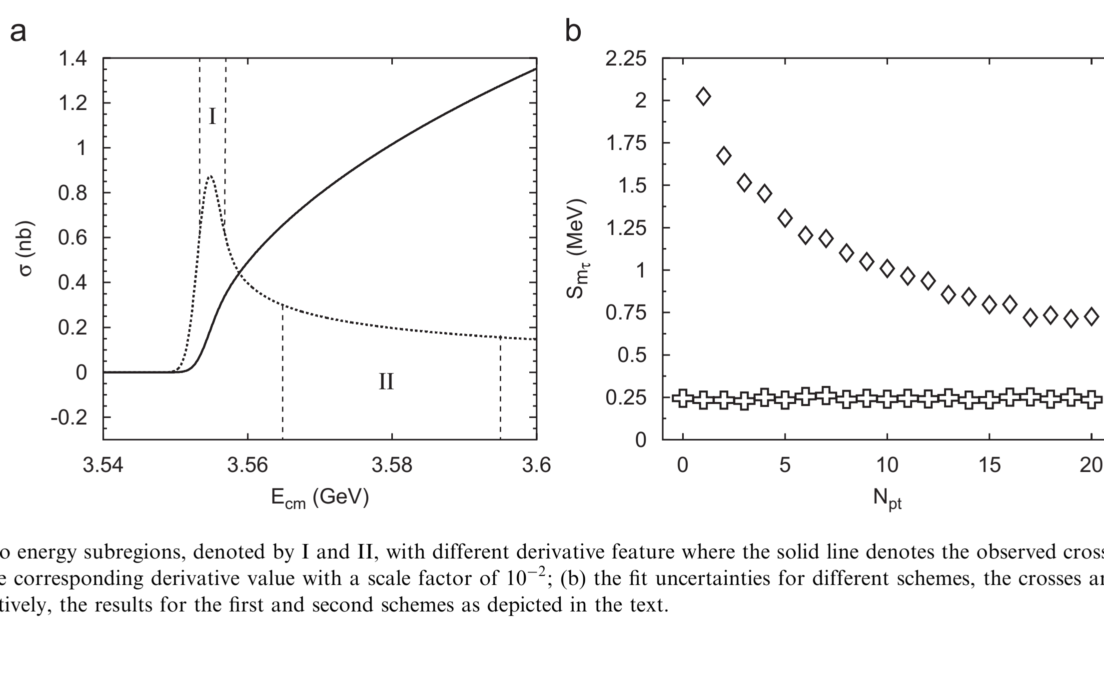

Fig. 4. (a) Two energy subregions, denoted by I and II, with different derivative feature where the solid line denotes the observed cross-section and the dashed line the corresponding derivative value with a scale factor of 10^{-2}; (b) the fit uncertainties for different schemes, the crosses and the diamonds denote, respectively, the results for the first and second schemes as depicted in the text.

study indicated, the small uncertainty is achieved near the peak of the derivative. If looking into the region from 3.5520 to 3.5565 GeV, it is found that the smallest $S_{m_\tau} = 0.105\,\mathrm{MeV}/c^2$ is obtained near the m_τ threshold ($E_{\mathrm{cm}} = 3.55398\,\mathrm{MeV}$), which has a deviation from the position ($E_{\mathrm{cm}} = 3.55484\,\mathrm{MeV}$) with the greatest cross-section derivative where $S_{m_\tau} = 0.122\,\mathrm{MeV}/c^2$. In addition, the study also indicates that within 2 MeV region the variation of S_{m_τ} is fairly smooth (from 0.105 to $0.127\,\mathrm{MeV}/c^2$), which is very favorable for actual data taking.

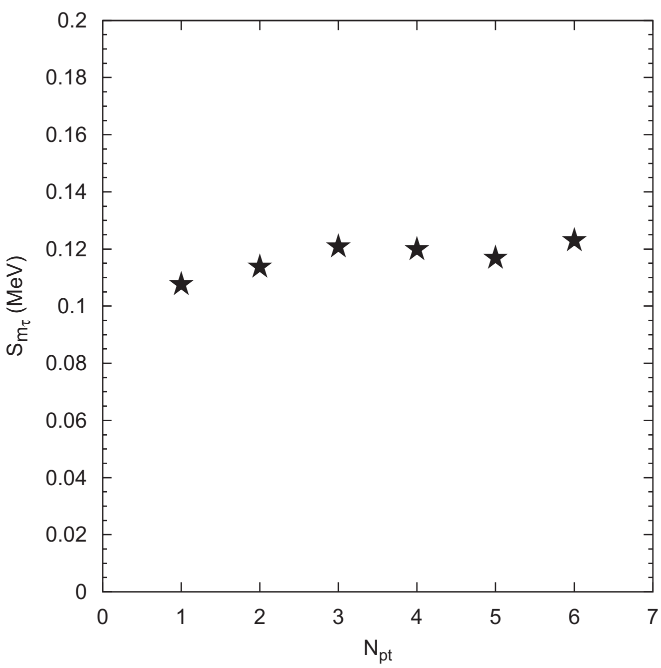

Fig. 5. The relation between S_{m_τ} and the number of points within the energy region from 3.553 to 3.557 GeV.

4.3. Luminosity and uncertainty

The last question we are concerned with is the relation between uncertainty and luminosity. For the fit with one point at the large derivative region, some special results are listed in Table 1. The second and third columns present the results for one point at $E_{\mathrm{cm}} = 3.55398\,\mathrm{GeV}$ which corresponds to the threshold while the last two columns give the results at $E_{\mathrm{cm}} = 3.55484\,\mathrm{GeV}$ which corresponds to the point with the greatest derivative of cross-section. In the light of Table 1, first the accuracy is inversely proportional to the luminosity; second the luminosity of $63\,\mathrm{pb}^{-1}$ is sufficient to provide the statistical precision less than $0.1\,\mathrm{MeV}/c^2$.

5. Discussion

At the forthcoming experiment BESIII the designed peak luminosity is around $1\,\mathrm{nb}^{-1}\,\mathrm{s}^{-1}$, if the average luminosity is taken as 50% of the peak value, then two days' data taking time will lead to the statistical uncertainty of m_τ less than $0.1\,\mathrm{MeV}/c^2$. Notice this evaluation is solely for eμ-tagged events, if more channels are utilized to tag τ-pair, such as ee, eμ, eh, μμ, μh, hh (h: hadron), and so on, more statistics can be expected (according to previous BES analyses [6,8] experience, the number of multi-channel-tagged events is at least 5 times more than that of eμ-tagged events). Therefore at BESIII, one week's data taking time will lead to statistical uncertainty less than $0.017\,\mathrm{MeV}/c^2$ for m_τ measurement.

For present fitting scheme, the systematic uncertainties due to various experimental factors are quantified by reasonable variations of the corresponding quantities (2% variation for luminosity $\mathscr{L}$, 2% for efficiency ε, 0.5% for branching fraction $B_{e\mu}$, 10% for background σ_{BG}, 30% for energy spread). In addition, the effect due to theoretical

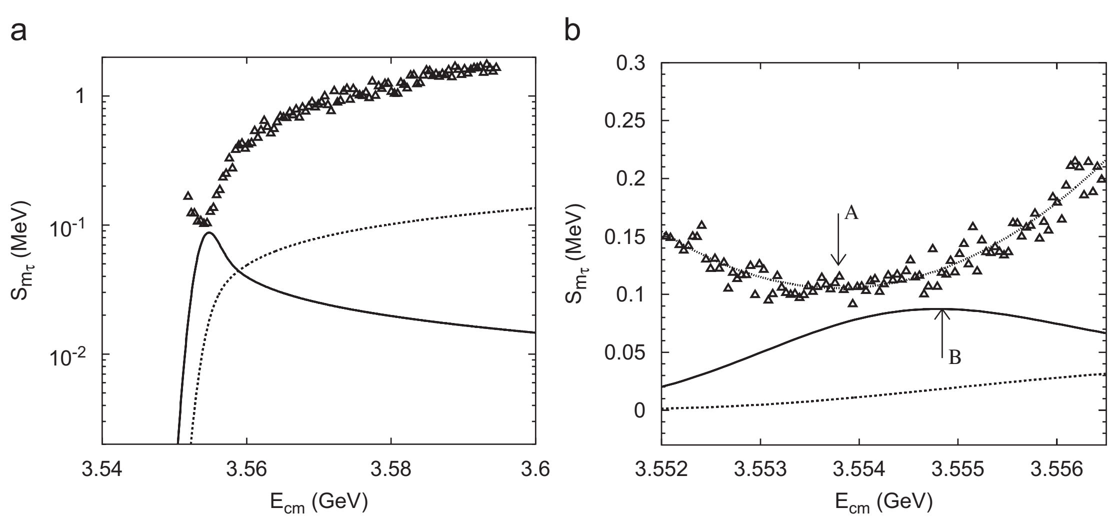

Fig. 6. The variation of S_{m_τ} against energy from one point fit with $\mathscr{L}_{\mathrm{tot}} = 45\,\mathrm{pb}^{-1}$. In plot (a) the scan region is from 3.551 to 3.595 GeV while in plot (b) the scan region is from 3.5520 to 3.5565 GeV. The solid line denotes the derivative of cross-section with scale factor 10^{-3} and the dashed line the observed cross-section with scale factor 10^{-1}.

Table 2
Systematic uncertainties for m_τ measurement

Source	δm_τ (10^{-3} MeV)	$\delta m_\tau/m_\tau$ (10^{-6})
Luminosity	14.0	7.9
Efficiency	14.0	7.9
Branching fraction	3.5	2.0
Background	1.7	1.0
Energy spread	3.0	1.7
Theoretical accuracy	3.0	1.7
Energy scale	100	56.3
Summation	102	57.5

calculation is by comparing two kinds of formulas with different accuracy [2,7,9]. The most important source of the uncertainty in m_τ measurement is the precision on the absolute beam energy determination. For the m_τ measurement at BESI, this uncertainty is 0.1 MeV/c^2 [13], which will be the dominant uncertainty if it is the same at BESIII, as summarized in Table 2. At present, new techniques can be used to determine the absolute energy value with an accuracy at the level of 10^{-5} or 10^{-6}. If such kinds of technique can be realized at BESIII, the systematic uncertainty due to energy scale can be greatly decreased.

At last, a few words about the data taking design for BESIII. To measure m_τ, data taking at three points will be desired. One is at the large derivative region, which is the optimal position as our study indicates; the other is below the threshold, which can be used for background study (to measure σ_{BG}); and the third one is above the threshold, which can be used to determine the event selection criteria for data analysis at the optimal point. In fact, the data below and above threshold can also be used for R-value measurement and background study for J/ψ and ψ' data analyses [12]. So it is reasonable to consider the m_τ measurement between the periods of data taking for the J/ψ and ψ'.

6. Summary

We adopt Monte Carlo simulation and sampling technique to simulate various data taking possibilities for a high precision measurement of τ mass, from which we find out the optimal scheme. As to the questions proposed in the Introduction, our answers are as follows:

1. The optimal position for data taking is located at the region near the $\tau^+\tau^-$ production threshold with large derivative of the $e^+e^- \to \tau^+\tau^-$ cross-section to energy.
2. One point is enough to achieve small error within the optimal region.
3. The luminosity of 63 pb^{-1} is sufficient for a statistical precision better than 0.1 MeV/c^2.

References

[1] A. Bogomyagkov, et al., Research of possibility to use beam polarization for absolute energy calibration in high-precision measurement of Tau Lepton mass at VEPP-4M, in: Proceedings of the 9th European Particle Accelerator Conference (EPAC 2004), Lucerne, Switzerland, July 5–9, 2004.
[2] M.B. Voloshin, Phys. Lett. B 556 (2003) 153.
[3] [a] B.H. Smith, M.B. Voloshin, Phys. Lett. B 324 (1994) 177;
[b] B.H. Smith, M.B. Voloshin, Phys. Lett. B 333 (1994) 564 Erratum.
[4] A. Pich, Nucl. Phys. Proc. Suppl. 169 (2007) 393 [hep-ph/0702074].
[5] BES Collaboration, Preliminary Design Report: The BESIII Detector, January, 2004.
[6] J.Z. Bai, BES Collaboration, et al., Phys. Rev. Lett. 69 (1992) 3021.
[7] J.Z. Bai, BES Collaboration, et al., HEP & NP 16 (1992) 343.
[8] J.Z. Bai, BES Collaboration, et al., Phys. Rev. D 53 (1996) 20.
[9] Y.K. Wang, X.H. Mo, C.Z. Yuan, J.P. Liu, HEP& NP 31 (2007) 325.
[10] W.-M. Yao, Particle Data Group, et al., J. Phys. G 33 (2006) 10.
[11] S. Brandt, Data Analysis, third ed., Springer, Verlag, 1999.
[12] P. Wang, C.Z. Yuan, X.H. Mo, HEP & NP 27 (2003) 456 (hep-ex/0210062).
[13] C. Zhang, et al., Beam energy stabilization and online measurement in BEOC, in: Proceedings of the VXth International Conference on High Energy Accelerators (HEACC'92 Hamburg), Hamburg, Germany, July 20–24, 1992.